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System of Nonlinear Ordinary Differential Equation Models of a Resource-Dependent Interacting Biodiversity in a Harsh Climate

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Abstract
This study develops and analyses a system of nonlinear ordinary differential equations to model the interaction between two competing leguminous species - cowpea and groundnut - under varying climate conditions and limited environmental resources. The model incorporates intrinsic growth, intra- and interspecific competition, resource consumption, and climate-induced stress factors. Numerical simulations were performed using the ODE45 solver to examine system dynamics under scenarios of no climate stress, as well as low, moderate, and severe climate conditions. Results indicate that increasing climate stress parameters significantly accelerates the decline and eventual extinction of both species, highlighting the critical role of environmental degradation in biodiversity loss. Conversely, resource availability increases over time as species populations diminish, reflecting reduced consumption pressure. A one-at-a-time sensitivity analysis further reveals that the system is highly sensitive to climate stress and competition parameters. The findings provide quantitative insight into the complex interplay between climate stress, species interaction, and resource dynamics, with implications for sustainable ecosystem management.

Keywords: Nonlinear Dynamical Systems, Biodiversity Modelling, Climate Stress, ODE45, Species Interaction

Introduction

Biodiversity plays a crucial role in maintaining ecosystem stability and agricultural productivity. However, increasing environmental stress due to climate change has significantly affected species survival and interaction patterns. Mathematical modelling provides a systematic framework for understanding such complex ecological dynamics (Atsu et al., 2017b; Bachelet et al., 2003; and Box, 1996). In agro-ecosystems, crops such as cowpea and groundnut compete for limited environmental resources, including nutrients, water, and space. These interactions become more critical under harsh climatic conditions, such as drought, heat stress, and soil degradation. Consequently, modelling these systems requires incorporating both biological interactions and environmental stressors (Chesson et al., 2008; Cramer et al., 1993; De Mazancourt et al., 2013; Dejene, 2018; Diaz et al., 2006). One way to understand interactions between two legumes, such as cowpea and groundnut, is to construct a deterministic mathematical model with a logistic structure. This model is defined by two intrinsic growth rate parameters, intra-species competition coefficients, initial conditions, and the length of the growing season (Ekaka-a et al., 2013; De Mazancourt et al., 2013; Ekaka-a et al., 2013; Mba et al., 2020; Mba et al., 2023; Mba & Okereke, 2022). The study of how parameter variation affects biodiversity is an active area of research. The notion of biodiversity loss has been identified as a major, devastating biological phenomenon that needs to be mitigated. Mba (2021), Ekaka-a (2009), and Ekaka-a et al. (2018) have utilised a MATLAB ode45 numerical scheme to quantify the effect of decreasing and increasing the inter-competition coefficient on biodiversity loss and gain, under the simplifying assumption of fixed initial conditions (4, 10). Following recent applications of numerical simulations to model biodiversity, Atsu et al. (2017a) observed that the mathematical technique of numerical simulation, which has rarely been applied to interpret the extent of biodiversity

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loss and biodiversity gain, is an important short- and long-term quantitative scientific process. A first-order ordinary differential equation is said to be linear in the independent and dependent variables if it can be expressed in the general form:

$\frac{dy}{dx} + P(x) = Q(x) \dots (1)$.

Where $P(x)$ and $Q(x)$are functions of (x) only.

Note that the coefficient of $\frac{dy}{dx}$ is unity, and that $y^1$and y appear linearly.

Generally, equations of the form:

$\frac{dy}{dx} + P(x)y = Q(x)y^n \dots (2)$ is non-linear if. $n > 1$

If, $n = 0$, equation (2) reduces to the form:

$\frac{dy}{dx} + P(x)y = Q(x) \dots (3)$. which is a linear first-order equation.

If, $n = 1$, then equation (2) can be written in the form:

$\frac{dy}{dx} + R(x)y = 0 \dots (4)$.

Where $R(x) = P(x) - Q(x)$.

Equation (4) is a variable separable equation.

If. $n > 1$, then equation (2) is non-linear. Jacob Bernoulli solved it as follows:

Step 1

Write equation (2) in the form:

$y^{-n}\frac{dy}{dx} + P(x)\, y^{1-n} = Q(x) \dots (5)$.

Step 2

Try the transformation:

$y^{1-n} = U(x) \dots (6)$.

Step 3

Then, differentiate both sides of equation (6) with respect to x:

$(1-n)y^{-n}\frac{dy}{dx} = \frac{du}{dx} \dots (7)$.

or

$y^{-n}\frac{dy}{dx} = \frac{1}{1-n}\frac{du}{dx} \dots (8)$.

Step 4

Equation (2) becomes:

$\frac{1}{1-n}\frac{du}{dx} + P(x)u = Q(x) \dots (9)$.

or

$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x) \dots (10)$.

Equation (10) is a first-order linear differential equation with the integrating factor $\mu = e^{-\int(1-n)P(x)dx}$.

Thus, the transformation (10) transforms Bernoulli's equation to a linear differential equation (Emenalo, 2013).

For the purpose of this study, the study considers systems of equation (2) when, $n > 1$ which has no closed-form solution; hence numerical solutions using ordinary differential equation solvers (ODE45) shall be required.

Materials and Methods

Mathematical Model Formulation

The physical model consists of two competing resources dependent biological interspecific species (cowpea and groundnut legumes) where both competing species are being affected by resources (soil nutrients, water, among others) in the habitat they are. To achieve the objectives of this study, a system of non-linear first order ordinary differential equations indexed by the appropriate initial condition as given stated by Mba (2021), has been extended by the present study. It is an ordinary differential equation because it involves derivatives with respect to a single independent variable.

where; C (t): population size of Cowpea in biomass, G (t): population size of Groundnut in biomass, and R (t): Resource availability (soil, nutrients, water, etc.)

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Model Equations with Harsh Climate effects are given by:

$\frac{dc(t)}{dt} = r_C C\left(1 - \frac{c}{K_C}\right) - B_{CG}CG - \propto_C C - \partial_C C,\ c(0) \geq 0$… (1)

$\frac{dG(t)}{dt} = r_G G\left(1 - \frac{G}{K_G}\right) - B_{GC}CG - \propto_G G - \partial_G G,\ G(0) \geq 0$… (2)

$\frac{dR(t)}{dt} = P - YR - \lambda_C CR - \lambda_G GR,\ R(0)) \geq 0$… (3)

Where:

$r_{C,}\ r_G$ → Intrinsic growth rates of cowpea and groundnut

$k_C\ , k_{G,}$, → Carrying capacities (maximum population sizes)

B_{CG}, B_{GC} → Competition coefficients

$\propto_C, \propto_G$ → (Climate stress parameter) Harsh climate effects (drought, heat stress, soil degradation, among others.)

∂_C, ∂_G → Mortality due to pests and diseases

P → Resource replenishment rate (rainfall, fertilization)

Y → Resource depletion rate due to environmental factors

λ_C, λ_G → Resource consumption rate of cowpea and groundnut legumes respectively.

Model Assumptions

1. The environment supplies a limited but renewable resource.
2. Both species compete for the same resource.
3. Climate stress increases mortality rates linearly.
4. Resource consumption is proportional to both population and resource level.

Special Case (No Climate Stress) where there is no harsh climate, we assume;

$\propto_C = \propto_G = 0$ (No climate stress)

$\partial_C = \partial_G = 0$ (Minimal pest/ disease mortality)

P = increases (better rain and soil health)

Numerical Method

Due to the nonlinear structure of the system, analytical solutions are not feasible. Therefore, the system was solved numerically using the **ODE45 solver in MATLAB**, which is based on the Runge–Kutta (4, 5) method.

Simulations were conducted under the following scenarios:

- No climate stress (α = 0)
- Low climate stress (α = 0.2)
- Moderate climate stress (α = 0.35)
- Severe climate stress (α = 0.64)

Results

Table 1: Quantifying the effect of increasing $\alpha_1 = 0.2\ and\ \alpha_2 = 0.2$ together on the yield of Cowpea and Groundnut using the ODE45 numerical method

T(weeks)	C(old)	C(new)	G(old)	G(new)	R(old)	R(new)
1.	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000
2.	27.9709	24.5533	1.5921	1.5105	0.3710	0.3609
3.	27.7155	20.9357	3.1934	2.8131	0.4097	0.5419
4.	27.5499	17.9783	3.2609	3.1896	0.4141	0.6713
5.	27.3886	15.4549	3.2436	2.8328	0.4165	0.7836
6.	27.2269	13.2832	3.2214	2.3698	0.4188	0.8984
7.	27.0646	11.4121	3.1989	1.9365	0.4212	1.0234
8.	26.9017	9.8007	3.1762	1.5605	0.4236	1.1616
9.	26.7382	8.4140	3.1535	1.2432	0.4261	1.3160

10.	26.5741	7.2216	3.1308	0.9791	0.4286	1.4879
11.	26.4094	6.1970	3.1079	0.7616	0.4312	1.6790
12.	26.2442	5.3170	3.0850	0.5840	0.4339	1.8922
13.	26.0784	4.5616	3.0620	0.4402	0.4367	2.1275
14.	25.9121	3.9132	3.0389	0.3249	0.4392	2.3888
15.	25.7454	3.3570	3.0158	0.2334	0.4416	2.6753
16.	25.5782	2.8798	2.9926	0.1282	0.4443	2.9903

Table 2: Quantifying the effect of increasing $\alpha_1 = 0.35\ and\ \alpha_2 = 0.35$ together on the yield of Cowpea and Groundnut using the ODE45 numerical method

T(weeks)	C(old)	C(new)	G(old)	G(new)	R(old)	R(new)
1.	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000
2.	27.9709	22.1275	1.5921	1.8114	0.3710	0.3884
3.	27.7155	16.9779	3.1934	1.8424	0.4097	0.6332
4.	27.5499	13.1101	3.2609	1.9779	0.4141	0.8607
5.	27.3886	13.1101	3.2436	1.5612	0.4165	1.1003
6.	27.2269	7.8315	3.2214	1.1286	0.4188	1.3728
7.	27.0646	6.0513	3.1989	0.7788	0.4212	1.6921
8.	26.9017	4.6750	3.1762	0.5174	0.4236	2.0678
9.	26.7382	3.6114	3.1535	0.3298	0.4261	2.5089
10.	26.5741	2.7897	3.1308	0.1994	0.4286	3.0203
11.	26.4094	2.1550	3.1079	0.1114	0.4312	3.6096
12.	26.2442	1.6648	3.0850	0.0538	0.4339	4.2790
13.	26.0784	1.2862	3.0620	0.0178	0.4367	5.0280
14.	25.9121	0.9938	3.0389	0.0036	0.4392	5.8530
15.	25.7454	0.7679	3.0158	0.0153	0.4416	6.7478
16.	25.5782	0.5934	2.9926	0.0182	0.4443	7.6984

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Table 3: Quantifying the effect of increasing $\alpha_1 = 0.64\ and\ \alpha_2 = 0.64$ together on the yield of Cowpea and Groundnut using the ODE45 numerical method

T(weeks)	C(old)	C(new)	G(old)	G(new)	R(old)	R(new)
1.	30.0000	30.0000	30.0000	30.0000	30.0000	30.0000
2.	27.9709	22.1275	1.5921	1.8114	0.3710	0.3884
3.	27.7155	16.9779	3.1934	1.8424	0.4097	0.6332
4.	27.5499	13.1101	3.2609	1.9779	0.4141	0.8607
5.	27.3886	13.1101	3.2436	1.5612	0.4165	1.1003
6.	27.2269	7.8315	3.2214	1.1286	0.4188	1.3728
7.	27.0646	6.0513	3.1989	0.7788	0.4212	1.6921
8.	26.9017	4.6750	3.1762	0.5174	0.4236	2.0678
9.	26.7382	3.6114	3.1535	0.3298	0.4261	2.5089
10.	26.5741	2.7897	3.1308	0.1994	0.4286	3.0203
11.	26.4094	2.1550	3.1079	0.1114	0.4312	3.6096
12.	26.2442	1.6648	3.0850	0.0538	0.4339	4.2790
13.	26.0784	1.2862	3.0620	0.0178	0.4367	5.0280
14.	25.9121	0.9938	3.0389	0.0036	0.4392	5.8530
15.	25.7454	0.7679	3.0158	0.0153	0.4416	6.7478
16.	25.5782	0.5934	2.9926	0.0182	0.4443	7.6984

Table 4: Quantifying the effect of decreasing $\beta_1\ and\ \beta_2$ together by 50% on the yield of Cowpea and Groundnut using the ODE45 numerical method

C(old)	C(new)	G(old)	G(new)	R(old)	R(new)
3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
2.3681	2.3865	1.1981	1.4333	2.1276	2.0142
1.8886	1.9081	0.5261	0.7223	3.0122	2.8006
1.5124	1.5294	0.2562	0.3863	3.9191	3.6579
1.2133	1.2274	0.1411	0.2218	4.8280	4.5565
0.9742	0.9857	0.0888	0.1380	5.7425	5.4855
0.7826	0.7920	0.0632	0.0932	6.6709	6.4381

Table 5: Quantifying the effect of decreasing β_1 and β_2 together by 70% on the yield of Cowpea and Groundnut using the ODE45 numerical method

C(old)	C(new)	G(old)	G(new)	R(old)	R(new)
3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
2.3681	2.3788	1.1981	1.3339	2.1276	2.0604
1.8886	1.8996	0.5261	0.6359	3.0122	2.8891
1.5124	1.5219	0.2562	0.3273	3.9191	3.7704
1.2133	1.2211	0.1411	0.1843	4.8280	4.6766
0.9742	0.9806	0.0888	0.1148	5.7425	5.6014
0.7826	0.7878	0.0632	0.0790	6.6709	6.5446

Table 6: Quantifying the effect of decreasing β_1 and β_2 together by 90% on the yield of Cowpea and Groundnut using the ODE45 numerical method

C(old)	C(new)	G(old)	G(new)	R(old)	R(new)
3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
2.3681	2.3715	1.1981	1.2417	2.1276	2.1055
1.8886	1.8921	0.5261	0.5603	3.0122	2.9724
1.5124	1.5153	0.2562	0.2778	3.9191	3.8721
1.2133	1.2157	0.1411	0.1540	4.8280	4.7811
0.9742	0.9762	0.0888	0.0965	5.7425	5.6994
0.7826	0.7842	0.0632	0.0678	6.6709	6.6327

C (old) represent the yield of Cowpea when all the model parameter values are fixed at 100%
G (old) represent the yield of Groundnut when all the model parameter values are fixed at 100%
R (old) represent the Resources generation when all the model parameter values are fixed at 100%
C (new) represent the yield of Cowpea when α_1 and α_2 , β_1 and β_2 are $varied$
G (new) represent the yield of Groundnut when α_1 and α_2 , β_1 and β_2 are $varied$
R (old) represent the yield of Resources when α_1 and α_2 , β_1 and β_2 are $vaied$
Numerical simulations reveal that increasing climate stress significantly affects species survival.

- Under low stress, populations decline gradually.
- Under moderate stress, extinction occurs more rapidly.
- Under severe stress, both species approach extinction quickly.

Conversely, resource levels increase as species populations decrease due to reduced consumption.
Tables 1–6 present detailed numerical outputs of the simulations.
G(old) represent the yield of Groundnut when all the model parameter values are fixed at 100%

R(old) represent the Resources generation when all the model parameter values are fixed at 100%

C(new) represent the yield of Cowpea when α_1 and α_2 , β_1 and β_2 are $varied$

G(new) represent the yield of Groundnut when α_1 and α_2 , β_1 and β_2 are $varied$

R(old) represent the yield of Resources when α_1 *and* α_2 , β_1 *and* β_2 *are vaied*

Discussion

In this study, we developed and analysed a system of nonlinear ordinary differential equations to model the dynamics of two interacting agricultural assets, cowpea and groundnut, along with a shared environmental resource called ecosphere assets. Table 1 to table 3 show the comparison of the absence of climate change ($\alpha_1 = 0$ *and* $\alpha_2 = 0$) with low climate change($\alpha_1 = 0.2$ *and* $\alpha_2 = 0.2$), mild climate change ($\alpha_1 = 0.35$ *and* $\alpha_2 = 0.35$) and severe climate change ($\alpha_1 = 0.64$ *and* $\alpha_2 = 0.64$) of cowpea and groundnut species. From the results obtained, higher increase of α_1 *and* α_2 together leads to faster extinction of cowpea and groundnut confirming that stronger degradation accelerate biodiversity loss. The resource level (R) increases significantly over time as few species remain to consume it.

When all the model parameter values are set; their default values, the system display typically predator-resource dynamics, both cowpea and groundnut species started off with population but eventually faced declines due to a mix variation of intra and inter-specific interactions. The value of cowpea C (new), and groundnut G (new) when α_1 *and* α_2 are varied together is less than the value when all the model parameters are fixed at 100% showing evidence of biodiversity loss.

Tables 4 to 6 show the effect of decreasing beta sub 1 and beta sub 2 by 50% to 90%, from the results obtained; as beta sub 1 and beta sub 2 decrease both cowpea and groundnut decline more rapidly. The values of cowpea C (old) and groundnut G (old) when all the model parameter values are at 100% are less than the values of cowpea C (new) and groundnut G (new) when β_1 *and* β_2 are varied alone showing evidence of biodiversity gain. The researchers conducted a one-at-a-time sensitivity analysis by varying key parameter values individually. Sensitivity analysis shows that:

- Climate stress parameters strongly influence extinction rates
- Competition coefficients affect coexistence stability
- Resource consumption rates determine long-term sustainability

These findings highlight the delicate balance required to maintain biodiversity in resource-limited environments. In summary, biodiversity loss is really affected by both direct and indirect factor. To manage resources sustainability, we need to find a balance between how much we exploit them e.g by controlling certain activity and reducing human caused mortality. Strong interactions between different agricultural assets can throw community dynamics off balance.

Conclusion

This study developed a nonlinear ODE model to investigate the interaction between cowpea and groundnut under climate stress. The results demonstrate that increasing environmental stress accelerates biodiversity loss while increasing resource availability due to reduced consumption. The model provides insights into sustainable ecosystem management and highlights the importance of controlling environmental stress and competition among species. Future work may extend the model to include stochastic effects, spatial dynamics, and additional species interactions.

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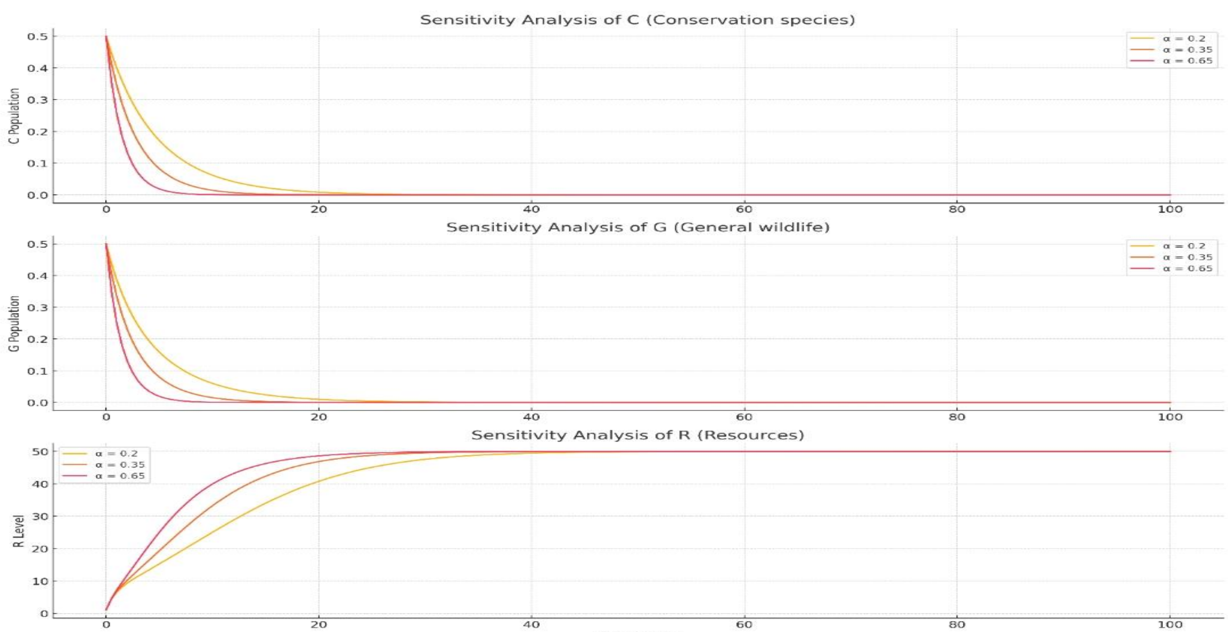
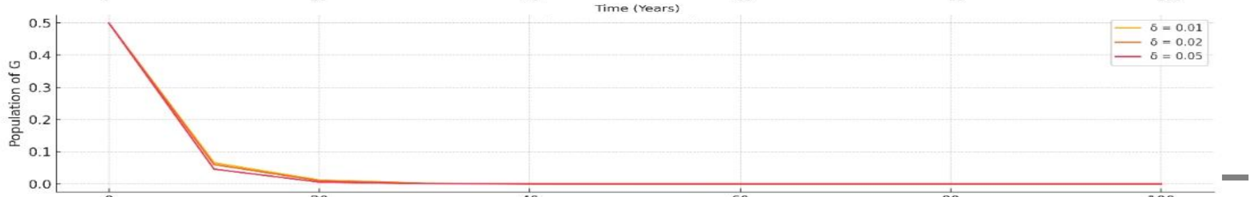
Appendices

Sensitivity Analysis





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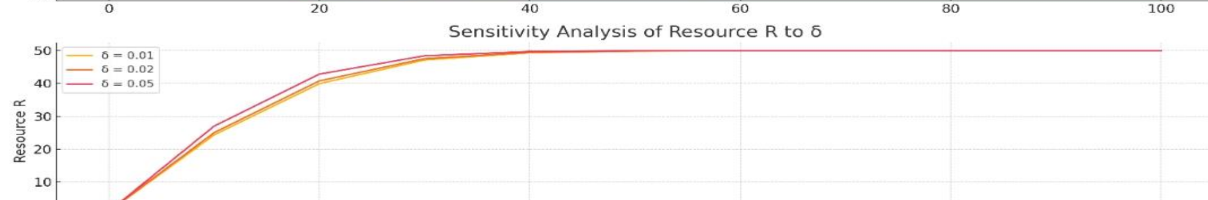

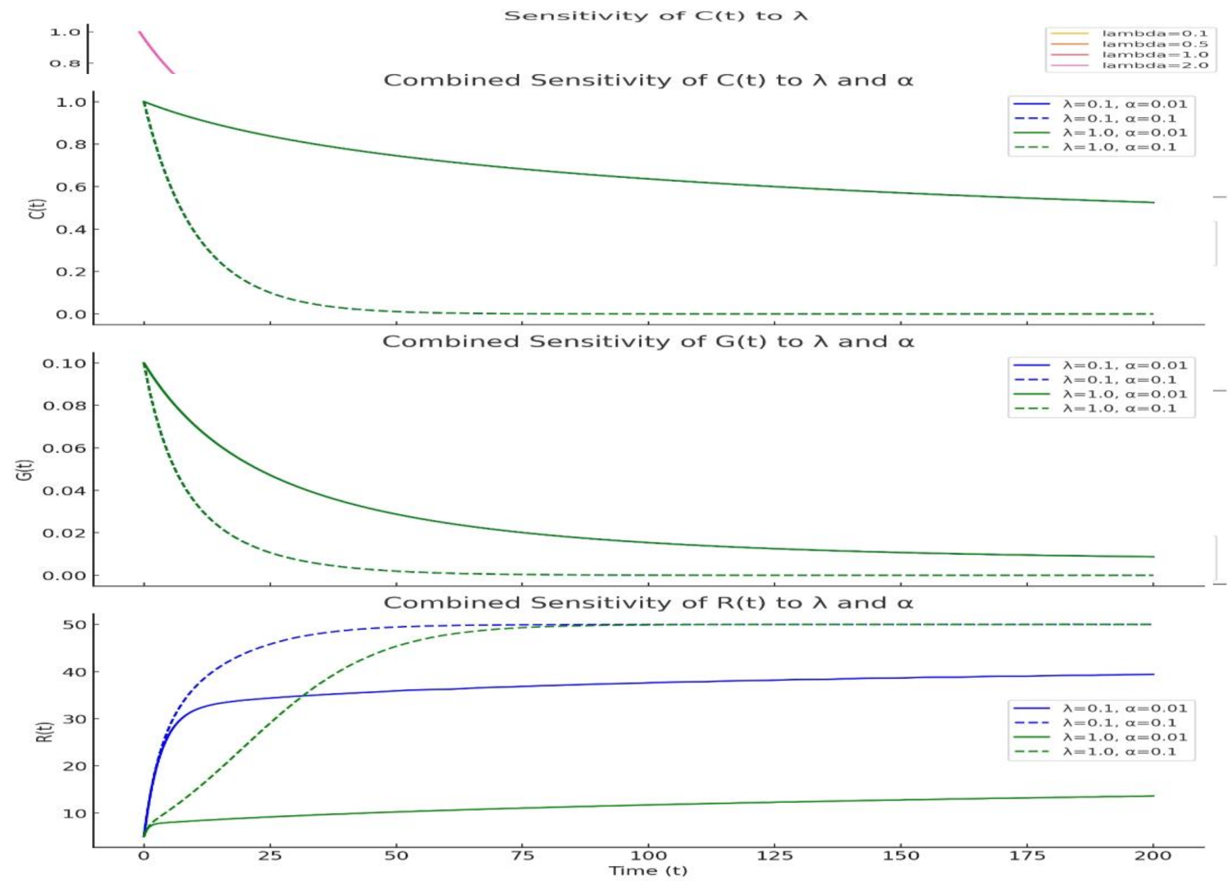
Sensitivity of C(t) to λ
lambda=0.1
lambda=0.5
lambda=1.0
lambda=2.0
Combined Sensitivity of C(t) to λ and α
C(t)
Combined Sensitivity of G(t) to λ and α
G(t)
Combined Sensitivity of R(t) to λ and α
R(t)
λ=0.1, α=0.01
λ=0.1, α=0.1
λ=1.0, α=0.01
λ=1.0, α=0.1
Time (t)