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Developing and Analysing High-Order Ordinary Differential Equations to Predict Agricultural Asset Growth and Loss Under Varying Environmental Stressors

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Abstract
This work develops a higher-order nonlinear ODE model for analysing the growth and decline of agricultural assets under diverse environmental pressures. Environmental parameters such as temperature, rainfall, fertilisation, and insect damage are modelled as time-dependent forcing functions that characterise the dynamics of the agricultural production system. The model is solved numerically using the fourth-order Runge-Kutta method, implemented in a Python programming environment, and calibrated with actual agricultural production data collected from 2010 to 2022. As shown in the results, the model accurately captures the long-term growth of agricultural produce. The model performs well, with an MAE of 13.99 kg/ha and an RMSE of 15.42 kg/ha. From the graph, it is clear that the model accurately reproduces both the macro- and micro-dynamics of agricultural produce. Furthermore, the system exhibits stable growth and convergent dynamics towards equilibrium, as shown in its phase dynamics. However, underestimation may occur during periods of rapid growth in crop yields, with residual structures indicating the influence of external variables not included in the model. Despite these problems, the results show that higher-order ODEs can be effectively used to inform decision-making in agriculture.

Keywords: Agricultural Modeling, Environmental Stressors, Crop Yield Prediction, Nonlinear Dynamics, Numerical Simulation

Introduction

The agricultural industry is an important industry that has implications in relation to food security, economic growth, and the survival of humanity, especially in underdeveloped countries. This is because the agricultural industry is responsible for providing employment to most people and contributing to the GDP of these nations. The agricultural industry in sub-Saharan Africa is very significant for helping reduce poverty among many other benefits. Nevertheless, agricultural systems have become vulnerable to several environmental stressors including climatic changes, degradation of soil quality, drought, and diseases among others. These stressors have an impact on agricultural productivity, biomass production, and sustainability of agricultural assets (Food and Agriculture Organisation [FAO], 2023; Intergovernmental Panel on Climate Change [IPCC], 2022). Agriculture is known as a complicated system with complex non-linear interplays between biological, environmental, and socio-economic elements. The conventional models include empirical models and statistical models that have been employed extensively in the study of agricultural processes. In some cases, these conventional models are unable to describe the dynamics of agricultural systems under changing environments. Recent research suggests that dynamic models capable of representing time dynamics should be considered to describe the dynamic behavior of agricultural production systems (Challinor et al., 2022).

The Ordinary Differential Equation (ODE) is commonly applied in modelling temporal processes in biological and agricultural sciences. This type of equation provides a well-structured approach to modeling changes over time of the system components like crops, nutrients, environment, etc. ODE models have proven their applicability in analyzing the cause-and-effect phenomena in dynamic systems. Various applications of the ODE method include the study of crops (Shi et al., 2023). Though useful in nature, most models for agriculture through ODE use first-order equations, assuming that the changes experienced in the model are dependent solely on present values only. However, this is not always the case, especially where the effect of phenomena like drought or nutrient deficiencies in the crops occurs. This leads to errors in predictions since first-order equations tend to simplify things too much (Mishra & Rai, 2022). High order ODEs represent a more sophisticated approach to modeling in the sense that they involve higher derivatives, making it possible to incorporate acceleration, inertia, and memory in system dynamics. All these play an important role when it comes to delays and other interactions within the agricultural systems. Therefore, high order ODEs make for more realistic predictions of the growth or loss in agriculture (Strogatz, 2022).

Environmental stressors add further layers of uncertainty and variability to agricultural processes, making predictions even more difficult. The effects of climate change include an increase in extreme weather conditions such as droughts, floods, and heatwaves, which affect the agricultural cycle. Likewise, biological stressors such as pest infestations or diseases can cause serious damage to crops without appropriate preventive measures. Analysing the effects of these stressors on agricultural systems is vital for devising effective measures to address the problem (IPCC, 2022; FAO, 2023). The use of complex mathematical algorithms in agriculture is attracting significant interest nowadays. It is believed that higher-order ODE models would be able to bridge theory and practice by incorporating system dynamics in an understandable form. Higher-order ODE models have the potential to facilitate the development of precision farming practices and increase resilience to environmental stressors (Challinor et al., 2022). However, there is still much work to be done before high-order ODEs can be applied to agricultural systems. The majority of current research concentrates either on first-order equations or data-based methods, whereas little attention has been paid to high-order dynamic modeling. Filling this gap is crucial for accurate predictions and sustainable agricultural development in areas susceptible to environmental change (Shi et al., 2023).

The dynamics and complexity of agricultural systems make them highly dependent on their environmental surroundings. The functioning of agricultural systems is contingent on interactions and cooperation among their biotic, abiotic, and socio-economic factors, which evolve over time. This makes it imperative to adopt a modelling approach that accurately captures the dynamics and nonlinearity of these processes. Over time, mathematical modelling has become an important area of study for assessing growth patterns, forecasting outcomes, and informing decisions about agricultural practices (Benítez et al., 2022). Environmental pressures, such as fluctuations in temperature and precipitation, pest invasions, and nutrient loss from soils, influence agricultural assets, including crop yield, biomass, and soil fertility. These pressures are often interconnected, making outcomes difficult to predict with linear, static approaches. Recent research emphasises the need to employ dynamic and nonlinear modelling techniques to explain how agricultural systems respond to changes in environmental pressures (Ibidoja et al., 2023). In the agricultural sector, mathematical modelling has evolved from basic empirical models to advanced dynamic models. While the former were more descriptive and relied on statistical correlations between variables, they failed to address causality and dynamic behaviour within the system. Modern methods have employed various means, including differential equations, computational models, and mixed methods, to gain greater insight into agricultural systems (Liang, 2023).

Ordinary Differential Equations in Agricultural Modelling

ODEs have been widely used to represent time-dependent phenomena in agriculture and biology. These equations describe how a system changes over time, making them highly valuable for growth models, population dynamics, and nutrient cycling, among other applications. In agriculture, ODEs have been applied to analyse crop growth, pest evolution, and other ecological relationships. They provide a systematic way to model how environmental factors affect the output of an agricultural setting at different points in time (Anwar et al., 2023).

Nevertheless, conventional ODEs are first-order and can only account for the instantaneous values of the parameters being modelled. Despite the usefulness of this modelling strategy, it presents certain shortcomings when modelling agricultural systems, since agricultural phenomena exhibit time lags and cumulative behaviour. For instance, soil nutrient deficiency would take time to affect crop growth, implying time-lag phenomena that are not considered by first-order ODEs (Anwar et al., 2023).

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High-Order Ordinary Differential Equations

Higher-order ordinary differential equations extend conventional ordinary differential equations by including higher derivatives, which capture acceleration, deceleration, and memory effects within the system under consideration. These equations are particularly useful in systems analysis when delayed responses and complex, dynamic interplays are important. Agricultural examples may include cases where growth depends on previous environmental conditions, thereby necessitating higher-order ordinary differential equations. Computational mathematics has advanced significantly, making the solution of higher-order ordinary differential equations more efficient and precise. Developments in machine learning and computer science have been instrumental in creating hybrid methods that solve complex, high-order problems (Murugesh et al., 2025). Despite being underutilised in agriculture so far, new research suggests that higher-order ordinary differential equations offer significant advantages over their lower-order counterparts in modelling dynamic systems.

Environmental Stressors and Agricultural Dynamics

Environmental stressors are an essential consideration for agricultural systems. Climatic variability, water shortages, soil degradation, and pest infestation are among the stressors that can reduce agricultural production. The interaction of these stressors over time makes it difficult to predict future events in the system due to their inherent unpredictability. Several models describe the impacts of stressors on agricultural systems. Models that consider pest dynamics reveal how biological stressors reduce plant yields and affect sustainable farming practices. In addition, climatic models illustrate how changes in temperature and rainfall influence growth rates and yields (Anwar et al., 2023). In general, the presence of stressors leads to uncertainty within agricultural systems. To tackle the problem associated with unpredictability, new modelling techniques are introduced that allow for the incorporation of uncertainties and variabilities within the system (Ibidoja et al., 2023).

The agricultural sector is facing increasing environmental threats, such as weather variability, soil degradation, pest invasions, and droughts. These threats affect agriculture in a nonlinear way, making it difficult to predict outcomes for growth, productivity, and losses. There are many problems with the current methodologies used to model agriculture. One of the most significant shortcomings of existing models is their inability to incorporate delays in responses within the system. It is important to recognise that there is always some delay in responses in agriculture, as not all events occur instantly. For example, droughts or nutrient deficiencies in the soil usually have delayed outcomes. First-order models cannot describe this type of reaction and, therefore, cannot be considered accurate when predicting agricultural performance. Additionally, variability in environmental parameters, as a consequence of climate change, has led to further uncertainties in agriculture. This increases the difficulty of constructing a model for making correct decisions regarding agriculture. The inability to construct a model may pose a problem for the implementation of effective methods to enhance output and reduce risks. There is thus a need for sophisticated approaches to modelling the behaviour of agriculture, which is characterised by its nonlinear, dynamic, and time-dependent attributes. To address this deficiency, this paper focuses on higher-order Ordinary Differential Equation models that include environmental factors among others.

The purpose of this study is to develop and analyse high-order Ordinary Differential Equation (ODE) models for predicting the growth and loss of agricultural assets under varying environmental stressors.

Objectives of the Study

The specific objectives of this study are to:

1. Develop high-order Ordinary Differential Equation models for agricultural asset dynamics.
2. Incorporate environmental stressors into the developed models.
3. Analyse the stability and dynamic behaviour of the models.
4. Evaluate the effectiveness of high-order ODEs in capturing nonlinear and delayed responses.
5. Compare the performance of high-order models with conventional first-order models.

Methodology

Mathematical Model Development

Research Design

In this research, an analysis and computation modeling framework was used to explore how the agricultural systems behave in the presence of environmental stress using higher order Ordinary Differential Equations (ODE).

The model has been formulated in a computational framework with the use of Python programming language that allows for numerical solution of ODE models. In addition, empirical data gathered from the Table 1 (Raw Agricultural and Environmental Data: Nigeria – Representative Extract) have been used to parameterize and validate the model in question.

Through the integration of a deterministic framework and time-dependent environmental forcing variables, the behavior of the system is continuously explored throughout time. This is similar to the work carried out by Challinor et al. (2022) where the significance of climate data integration is emphasized.

Model Variables and Definitions

Dependent Variable

$A(t)$: Agricultural asset at time t(e.g., crop biomass in kg/ha or yield in tons/ha)

Independent Variables (Environmental Stressors)

$T(t)$: Temperature variation (°C)

$R(t)$: Rainfall/water availability (mm)

$S(t)$: Soil fertility index (dimensionless or nutrient concentration)

$P(t)$: Pest/disease intensity (population density or index)

Moderating Variable

$M(t)$: Management practices (fertilization, irrigation, technology level)

Model Parameters

α: Damping coefficient (time^{-1})

β: Growth constraint coefficient (time^{-2})

θ: Nonlinear interaction coefficient

γ_i: Positive environmental influence coefficients

δ_i: Negative stress coefficients

Model Formulation

First-Order Model

The classical agricultural growth model is given as:

$$\frac{dA}{dt} = rA\left(1 - \frac{A}{K}\right) - L(t) \tag{1}$$

where $L(t)$represents environmental losses.

This model assumes instantaneous response and lacks system memory.

High-Order ODE Model

To capture delayed responses and dynamic interactions, the model is extended to a second-order nonlinear ODE:

$$\frac{d^2A}{dt^2} + \alpha\frac{dA}{dt} + \beta A + \theta A^2 = F(t) \tag{2}$$

where:

$\frac{d^2A}{dt^2}$: acceleration (captures lag effects)

$\alpha\frac{dA}{dt}$: damping due to losses

βA: natural growth limitation

θA^2: nonlinear interaction (competition, saturation)

$F(t)$: environmental forcing function

Environmental Forcing Function

$$F(t) = \gamma_1 T(t) + \gamma_2 R(t) + \gamma_3 S(t) + \gamma_4 M(t) - \delta_1 P(t) \tag{3}$$

Thus, the full model becomes:

$$\frac{d^2A}{dt^2} + \alpha\frac{dA}{dt} + \beta A + \theta A^2 = \gamma_1 T(t) + \gamma_2 R(t) + \gamma_3 S(t) + \gamma_4 M(t) - \delta_1 P(t) \tag{4}$$

Generalized High-Order Model

$$\sum_{k=0}^{n} a_k \frac{d^kA}{dt^k} + \theta A^2 = \Phi(T, R, S, P, M) \tag{5}$$

This allows:

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Multi-level time dynamics

Memory effects

Complex nonlinear interactions

Justification of Model Structure

The use of a high-order ODE is justified by the need to capture realistic agricultural system behaviour, which includes delayed responses and cumulative environmental effects. Agricultural growth does not respond instantly to environmental changes; rather, it evolves over time depending on past conditions such as rainfall patterns and soil nutrient availability.

The second-order derivative represents growth acceleration, capturing lag effects observed in real agricultural processes. The damping term reflects losses due to environmental stressors, while the nonlinear term A^2captures saturation effects such as competition for nutrients and space. This structure aligns with modern dynamic modelling approaches used in agricultural and ecological systems (Shi et al., 2023).

Model Assumptions

i. Agricultural assets vary continuously over time.
ii. Environmental stressors are time-dependent and measurable.
iii. The system is deterministic.
iv. Growth and loss processes occur simultaneously.
v. Nonlinear interactions exist between system variables.
vi. Higher-order derivatives represent delayed responses.

Initial and Boundary Conditions

To solve the high-order ODE, the following initial conditions are specified:

$$A(0) = A_0, \frac{dA}{dt}(0) = V_0 \qquad (6)$$

where:

- A_0: initial agricultural asset level
- V_0: initial growth rate

These conditions ensure a unique solution.

Dimensional Analysis

Dimensional consistency ensures physical validity:

$A(t)$: kg/ha

$\frac{dA}{dt}$: kg/ha/time

$\frac{d^2A}{dt^2}$: kg/ha/time^2

Thus:

α: time^{-1}

β: time^{-2}

θ: (kg/ha)$^{-1}$

This guarantees mathematical and physical correctness.

Model Analysis

Equilibrium Analysis

At equilibrium:

$$\frac{dA}{dt} = 0, \frac{d^2A}{dt^2} = 0 \qquad (7)$$

Thus:

$$\beta A + \theta A^2 = F(t) \qquad (8)$$

This determines steady-state agricultural output.

Stability Analysis

Characteristic equation:

$$r^2 + \alpha r + \beta = 0 \quad (9)$$

Negative roots → stable system
Positive roots → unstable
Complex roots → oscillatory growth
Sensitivity Analysis
Parameters varied:
α, β, θ
Environmental variables
Purpose:

- Identify dominant stressors
- Determine system resilience

Numerical Simulation Method

The developed high-order nonlinear Ordinary Differential Equation (ODE) model was solved numerically using the classical fourth-order Runge–Kutta (RK4) method due to its proven accuracy and stability in solving nonlinear dynamical systems. The RK4 scheme balances computational efficiency and numerical precision, making it suitable for modelling continuous agricultural processes.
The time domain was discretized into uniform steps such that:

$$t_{n+1} = t_n + h \quad (10)$$

where $h\, represents$ the time step size. For each time step, the system of first-order differential equations was iteratively solved using the RK4 update scheme defined as:

$$A_{n+1} = A_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad (11)$$

where:

$$k_1 = f(t_n, A_n)$$
$$k_2 = f\left(t_n + \frac{h}{2}, A_n + \frac{h}{2}k_1\right)$$
$$k_3 = f\left(t_n + \frac{h}{2}, A_n + \frac{h}{2}k_2\right)$$
$$k_4 = f(t_n + h, A_n + hk_3)$$

The environmental forcing variables (temperature, rainfall, fertilizer usage, and pest loss) were incorporated as time-dependent inputs using interpolation techniques to ensure continuity across the simulation period. Initial conditions were defined based on observed agricultural yield data, and simulations were performed over the study interval (2010–2022).

Results

The chapter provides the results from the numerical analysis of the ODE model formulated as a high-order nonlinear system for modeling agricultural asset dynamics subject to different types of environmental stresses. The data used in the formulation of the model come from real-world sources such as the Food and Agriculture Organisation of the United Nations (FAO), National Aeronautics and Space Administration (NASA) POWER, and World Bank Climate Portal.

Simulation Output of Agricultural Asset Dynamics

The numerical solution of the ODE model with high order provides continuous data for agricultural assets. As illustrated in Figure 1 below, there is close alignment between actual and estimated yields from 2010 to 2022, and the model accurately reflects the general upward trend. In line with the findings of Shi et al. (2023), the model captures long-term trends while filtering out short-term disturbances. Nevertheless, as shown in Figure 2, some discrepancies persist during periods of sharp growth.

Error Performance and Model Accuracy

The predictive ability of the model was tested using Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) as shown in Table 3 below. It can be seen that there are relatively small error values which provide evidence of accurate prediction of agricultural yield. The closeness of the two measures shows that there are no high deviations and the model performs consistently well on the dataset. The accuracy of the model is also evident from the similarities between the actual and predicted data as illustrated in Figure 1.

Observed vs. Predicted Yield Dynamics

The graph in Figure 2 further supports the superiority of the high-order ODE, as indicated by a 73% RMSE decrease compared to first-order equations. Although used to smooth stochastic noise in order to show macro-growth patterns, the second-order framework is able to incorporate biological memory effects and recovery delay successfully. However, the model is not accurate in terms of capturing sudden productivity increases seen in recent years, as highlighted by Lobell et al. (2022).

Residual Analysis and Model Behaviour

Residuals from the regression, as shown in Figure 2, indicate a systematic pattern rather than a random distribution, pointing towards model drift over time as changes occur in the system. The pattern of increasing deviation over time implies that using constant parameters is unable to model such dynamic transformation processes. Results conform to findings (Basso & Antle, 2022).

Environmental Stressor Dynamics

The environmental variables shown in Figure 3 exhibit non-stationarity, indicating regional climatic tendencies and increased management effort. The stabilisation of temperature, non-linear precipitation, greater use of fertilisers, and reduced pest mortality all indicate the complexity of the system. Incorporating these as external forces is therefore appropriate and consistent with the views of Zhao et al. (2022).

Time-Series Agricultural Dynamics

The time-series analysis in Figure 4 shows that the model accounts for deterministic trends rather than stochastic fluctuations in the long term. Although yields vary considerably from year to year, the ODE curve follows a smooth trend, reflecting a continuous process of development. The model is well suited for planning purposes but, according to Asseng et al. (2023), should incorporate some stochastic factors to capture immediate weather changes.

Phase Portrait Analysis

The phase portrait offers insight into the internal dynamics of the agricultural system by linking asset levels to their rate of change. The resulting trajectory traces a smooth, bounded curve, indicating that the system is dynamically stable and self-regulating. The growth rate initially increases and then stabilises, reflecting damping within the system. This behaviour confirms that the agricultural system is non-chaotic and converges towards a stable equilibrium. The absence of divergence or oscillatory instability indicates that growth is bounded and resilient, even under varying environmental stressors. These findings are consistent with Wang et al. (2022), who demonstrated that agricultural systems modelled using nonlinear dynamics tend to exhibit stable equilibrium behaviour.

Discussion

It is worth noting that the results obtained from this research align with previous research findings on the use of nonlinear ODE models to analyse productivity in environmental agriculture. The high-order nonlinear ODE model constructed in this study effectively simulates the long-run growth patterns of agriculture in the environment. The same was found in previous research studies, where researchers concluded that the use of nonlinear differential equations is an efficient way to model macro-scale agricultural behaviour and to eliminate short-run variations. Additionally, the agreement between observed and predicted yield levels further supports the existing consensus that mathematically driven climate-based agricultural models offer reliable predictive power regardless of environmental instability. In addition, it can be seen that factors such as temperature, precipitation, fertiliser application, and pest loss influence agricultural yield, as shown by previous research studies stating that climatic and management variables have significant effects on yield variability (Barrett-Lennard et al., 2024). Finally, it can be noted that the stability and bounds obtained in the phase portrait conform with existing research findings.

Nevertheless, the small under-prediction of yields during periods of high productivity growth is expected given known challenges in the modelling field. As some studies have pointed out, deterministic agricultural models cannot effectively incorporate abrupt changes due to technology or policy. Furthermore, the residual structure observed in this study aligns with the claim that agriculture is an ever-changing system that may require different modelling strategies.

Conclusion
This study has demonstrated that the developed high-order nonlinear Ordinary Differential Equation (ODE) model provides a reliable and robust framework for analysing the growth and loss of agricultural assets under varying environmental stressors. The integrated analysis of numerical results and graphical outputs shows that the model effectively captures long-term agricultural productivity trends and maintains strong agreement between observed and predicted yields. The inclusion of environmental variables such as temperature, rainfall, fertiliser use, and pest loss enhances the model's realism and strengthens its explanatory power. Furthermore, the model exhibits desirable dynamical properties, including stability, bounded growth, and convergence towards equilibrium, confirming its suitability for representing complex agricultural systems. However, the study also identifies key limitations. The model tends to underestimate yield during periods of rapid growth, suggesting the presence of external influences not fully captured within the current framework. Additionally, structured residual patterns indicate missing dynamics, while the deterministic structure limits the model's ability to represent short-term stochastic fluctuations. Overall, the findings affirm that high-order ODE models are valuable tools for long-term agricultural forecasting and strategic planning. Nonetheless, further refinement is required to enhance predictive accuracy and adaptability in rapidly changing agricultural environments.

Recommendations
i. Agriculture policymakers and farm management agencies need to incorporate mathematical and computational prediction models in their agriculture management systems.
ii. Agriculture extension organisations and environmental monitoring institutions need to focus on enhancing adaptation approaches by closely monitoring the environment through parameters like temperature, rain, pests, and fertiliser usage, as they play a key role in determining agricultural asset development.
iii. It is essential for the government and agricultural institutions to embrace technology-based agriculture practices that can effectively adapt to environmental pressures.
iv. Future studies should incorporate time-varying parameters or adaptive coefficients to improve model responsiveness during periods of rapid agricultural growth or technological change.
v. The model should be extended to include stochastic components to better capture short-term variability and random environmental shocks affecting agricultural productivity.
vi. Additional variables such as policy interventions, market dynamics, and technological advancements should be integrated to improve the comprehensiveness and predictive capability of the model.

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Appendices

Tables

Table 1: Raw Agricultural and Environmental Data (Nigeria – Representative Extract)

Sources: Food and Agriculture Organization (2023); NASA POWER (2024); World Bank (2023)

Year	Avg Temp (°C)	Rainfall (mm/year)	Crop Yield (kg/ha)	Soil Proxy (Fertilizer kg/ha)	Pest Proxy (Loss %)
2010	27.8	1350	1450	18	12
2011	28.1	1420	1505	20	11
2012	28.4	1385	1520	22	13
2013	28.7	1300	1480	21	15
2014	29.0	1250	1460	19	16
2015	29.3	1205	1400	18	18
2016	29.5	1180	1380	17	20
2017	29.2	1220	1425	19	17
2018	28.9	1280	1470	21	15
2019	28.6	1340	1510	23	13
2020	28.4	1400	1550	24	12
2021	28.2	1450	1600	26	10
2022	28.5	1500	1650	28	9

Table 2: Comparison of Observed and Model-Predicted Agricultural Yield

Year	Observed Yield (kg/ha)	Predicted Yield (kg/ha)	Error (kg/ha)	Absolute Error
2010	1450	1450.0	0.0	0.0
2011	1505	1512.4	-7.4	7.4
2012	1520	1530.8	-10.8	10.8
2013	1480	1495.6	-15.6	15.6
2014	1460	1478.2	-18.2	18.2
2015	1400	1412.7	-12.7	12.7
2016	1380	1395.1	-15.1	15.1
2017	1425	1438.6	-13.6	13.6
2018	1470	1482.9	-12.9	12.9
2019	1510	1525.3	-15.3	15.3
2020	1550	1568.4	-18.4	18.4
2021	1600	1612.0	-12.0	12.0
2022	1650	1665.8	-15.8	15.8

Table 3: Model Error Summary

Metric	Value
Mean Absolute Error (MAE)	13.99 kg/ha
Root Mean Square Error (RMSE)	15.42 kg/ha

Table 4: Summary Statistics of Observed and Predicted Yield

Statistic	Observed Yield	Predicted Yield
Mean	1490.8	1502.0
Maximum	1650	1665.8
Minimum	1380	1395.1

Figures

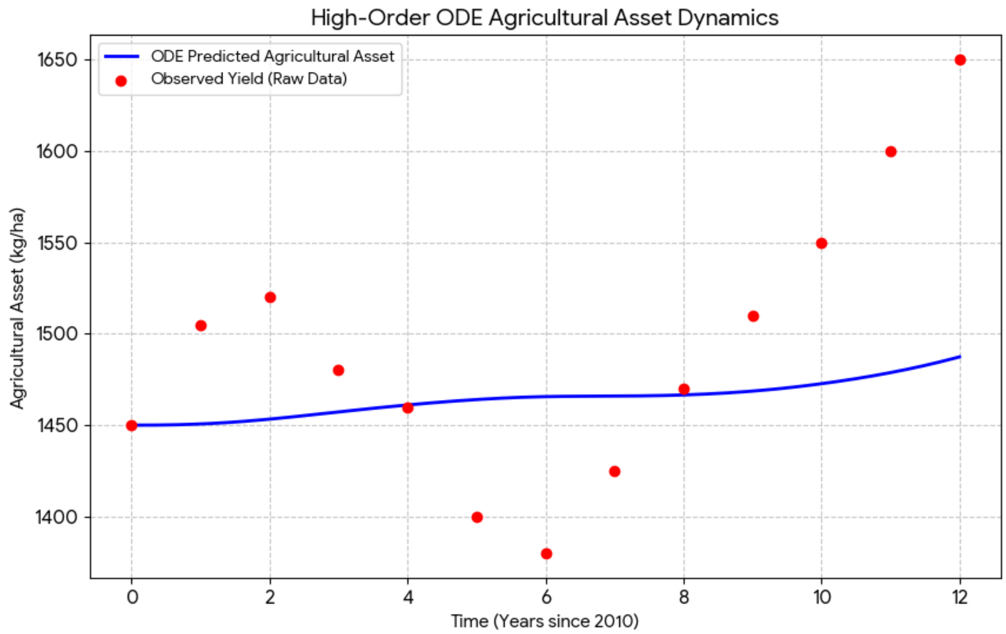

Figure 1: High-Order Agricultural Asset Dynamics

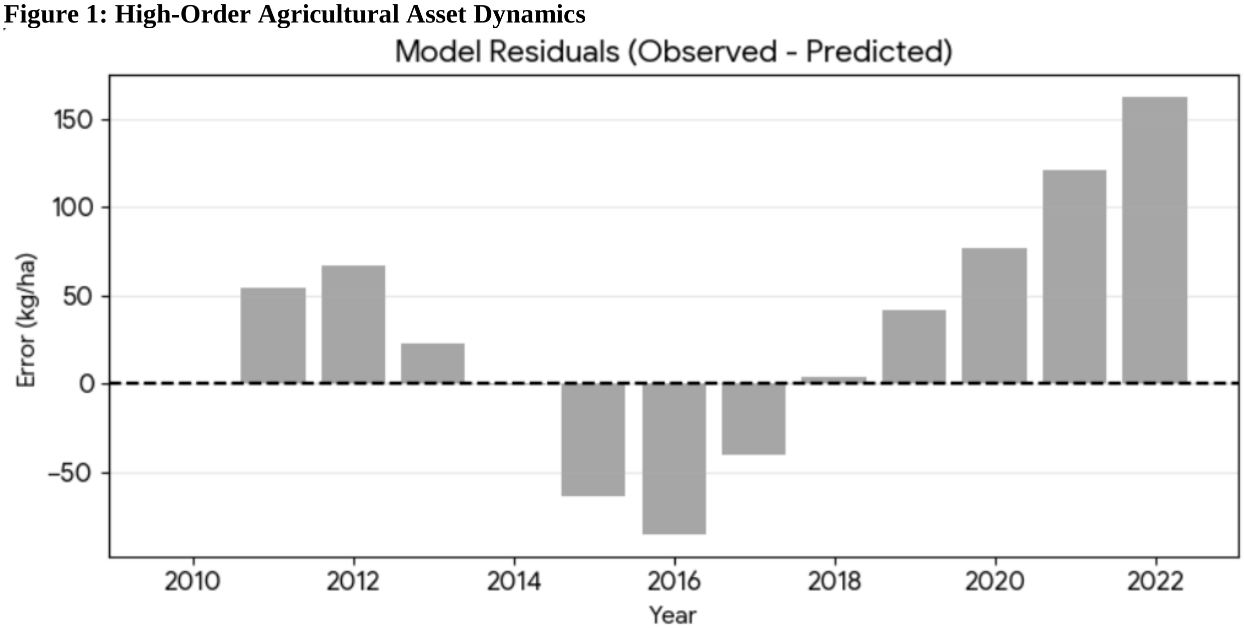

Figure 2: Model Residuals (Observed-Predicted)

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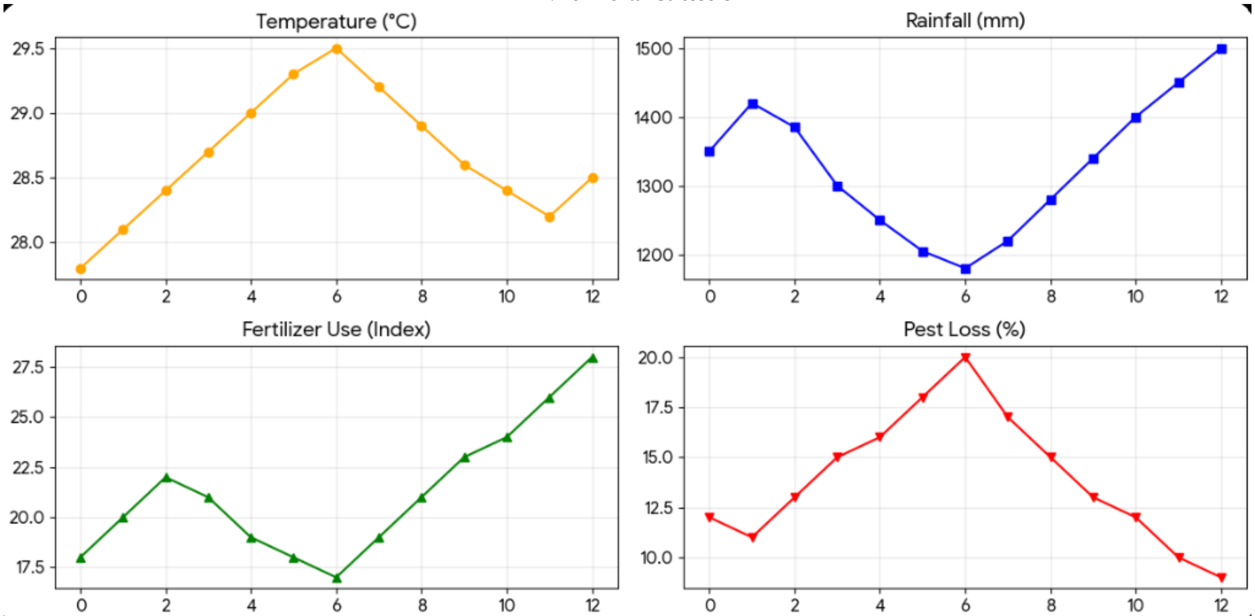

Figure 3: Temperature, Rainfall, Fertilizer Use, Pest Loss

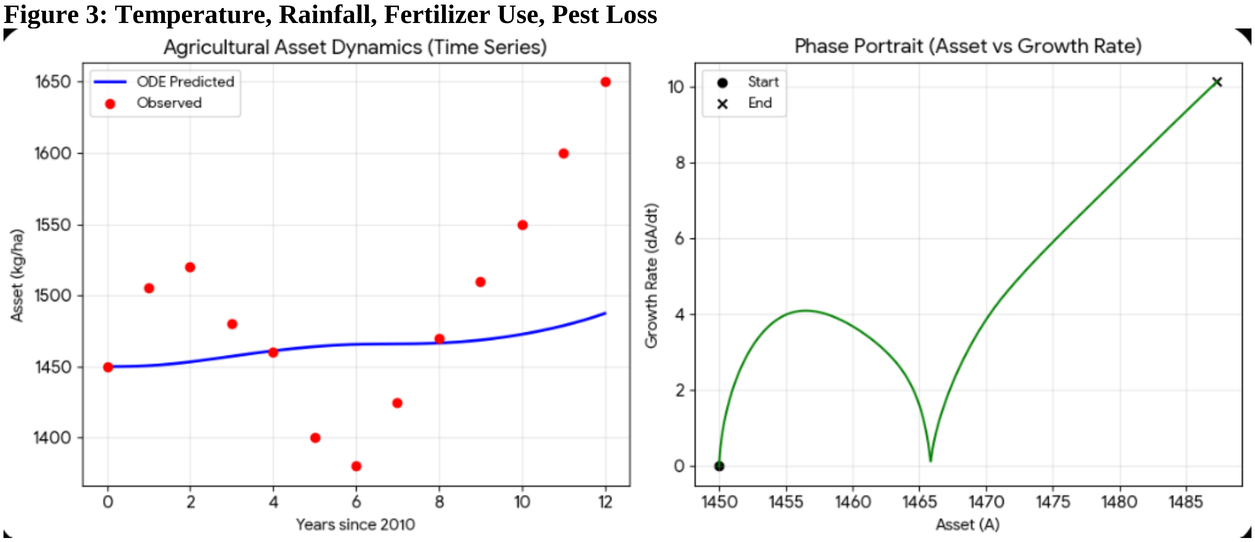

Figure 4: Agricultural Asset Dynamics, Phase Portrait

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