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Systematic Error Analysis in Technology-Enhanced Calculus: Comparing Maple and GeoGebra Approaches in Pre-Service Teacher Education in Ghana

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Abstract

The study focused on errors committed by 120 Ghanaian Pre-service teachers learning the volumes of solids of revolution (VOSR) through the lecture method or technology-enhanced instruction (TEI) using Maple and GeoGebra. A quasi-experimental pretest–posttest nonequivalent groups design was used. The errors were categorised according to Pólya's four problem-solving stages: understanding, planning, carrying out, and verification. The errors at pretest were dominated by those in verification (77.5%) and carrying out (70.0%), with no difference between the groups. Errors at pretest were dominated by those of verification (77.5%) and carrying out (70.0%), with no differences between groups. Technology group error reduction was significantly higher than that of the lecture instruction group after 8 weeks. Mean errors decreased by 78.3% for Maple (M pretest = 115, M posttest = 25) and 76.9% for GeoGebra (M pretest = 104, M posttest = 24), compared to 41.2% for lecture (M pretest = 114, M posttest = 67). Concept errors were reduced the most by Maple (88.0%), and procedural errors by GeoGebra (84.0%). For both, the number of calculation errors decreased (GeoGebra: 8.3%; Maple: 5.0%). The highest percentage of spatial reasoning errors was found in the group using GeoGebra (25.0%), and the verification errors were high across all groups, with significant differences between the lecture and the technology groups according to χ^2 analysis ($\chi^2(3, N = 116) = 15.23, p = .002, V = .58$) but not between Maple and GeoGebra.

Keywords: GeoGebra, Maple, Pólya, Technology-Enhanced, Calculus

Introduction

Studies show that examining student errors within the global mathematics education community provides essential insights into the development of mathematical concepts (Shimizu & Kang, 2025). Another study discusses the effectiveness of delivering mathematical concepts (Benecke & Kaiser, 2023). Contemporary research in the field of mathematics education views student errors as windows to the thought process of students (Riccomini, 2005), providing educators with information about misconception tendencies, incomplete schemas (Peng & Luo, 2009) and common error patterns that can be used to inform specific targeted pedagogical interventions (Bray, 2011). This perspective is especially useful in the study of calculus, where abstract concepts, complex procedures, and spatial reasoning pose significant challenges to learning across a multitude of educational settings (Nguyễn, 2015; Villavicencio, 2023). In the Teacher Education system in Ghana, where pre-service mathematics teachers consistently have difficulty learning advanced mathematical concepts (Anamuah-Mensah et al., 2008; Mereku, 2010), it is necessary to analyse these errors systematically to develop proper pedagogical interventions. The colleges of education in Ghana that provide calculus

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instruction experience various challenges, including class sizes averaging 50-70, limited instructional resources, prevalent lecture-based pedagogy, and the majority of students previously having a weak mathematical background in the senior high school system (Armah et al., 2018; Etsey et al., 2005). These contextual constraints, along with the very difficult nature of calculus concepts, create an error-prone environment for students (Mereku, 2010).

The consideration of solids of revolution is one of the most difficult topics in the College of Education (Akyeampong et al., 2013). Calculus draws on many different mathematical abilities. This includes an understanding of functions, algebraic manipulations of these functions, integration and visualisation in three-dimensional space (Grundmeier et al., 2006; Martin, 2000). For future teachers at Colleges of Education in Ghana, mastering solids of revolution is particularly critical (Akyeampong et al., 2013). The Ghana Tertiary Education Commission (GTEC, 2019) curriculum specifically identifies integrals that can be used to produce solids of revolution as part of the college calculus course. The purpose was to prepare future teachers to teach integrals as part of their senior school curriculum (GTEC, 2019; Akyeampong et al., 2013). In response to students' difficulties in solving solids of revolution, several researchers worldwide have examined the impact of integrating technology into calculus instruction for post-secondary students (Hohenwarter & Fuchs, 2004; Roschelle et al., 2010; Stewart, 2016).

Many argue that technology integration can help solve calculus learning problems (Roschelle et al., 2010). However, little systematic research exists on how different technological approaches affect specific error types in developing countries. For example, computer algebra systems (CAS), such as Maple (Lavicza, 2010), offer computational assistance and step-by-step procedural guidance, while dynamic geometry software, such as GeoGebra (Hohenwarter & Fuchs, 2004), supports interactive visualisation. These technologies focus on interactive visualisation, error reduction and calculus exploration (Roschelle et al., 2010). Despite the positive technological support in data-driven decisions, the literature remains silent in Ghana.

This study sought to investigate error patterns of 120 pre-service Calculus students selected from three instructional methods (traditional, Maple-enhanced and GeoGebra-enhanced). The study used both statistical (e.g., frequency) analysis of errors and qualitative analysis of error types to: 1) increase our knowledge of how technology shapes mathematical problem-solving; and 2) identify persistent error patterns that need to be further supported through instruction. This study provides evidence to fill in the 'gaps' identified by Fikru et al. (2024). To investigate all possibilities with specific errors made by calculus students, Pólya's problem-solving Framework became a valuable resource for this study (Burkhard et al., 2019). The four key stages for problem solvers included: (1) Understanding the Problem, (2) Developing a Plan, (3) Implementing the Plan, and (4) Reflecting on the Problem. The Pólya problem-solving Framework is a way for educators to diagnose and understand where students struggle while solving problems. This also includes the cognitive processes when students have difficulty solving problems (Lester, 2013). For students to understand the problem, they needed to recognise the available information. Students also needed to identify the problem and understand the vocabulary used in the problem (Burkhard et al., 2019). Students needed to mentally visualise the three-dimensional object from a two-dimensional representation (Burkhard et al., 2019).

In the case of VOSR problems, the first difficulty is understanding the calculus problem. This involves how students would sketch the 2D shape. Students need to rotate the shape about the axis to create the 3D solid from the two-dimensional representation. Many errors occur at this stage of problem solving (Ryan & Williams, 2007). Devising a plan involves selecting appropriate strategies. Students also need to determine relevant formulae or methods for the solution (Lester, 2013). When solving VOSR problems, students must decide between the disk or washer and shell methods, establish integration bounds, and determine the radius function (Schoenfeld, 2016). Planning errors frequently reflect inadequate conceptual understanding of when different methods apply and how they relate to problem structures (Schoenfeld, 2016). Carrying out the plan encompasses executing chosen strategies, performing algebraic manipulations, computing integrals, and managing procedural steps (Carlson & Bloom, 2005). Execution errors include algebraic mistakes, failures in integration techniques, and computational inaccuracies. While these errors appear procedural, they often reveal underlying conceptual gaps that prevent students from recognising when calculations diverge from correct paths (Ngcobo, 2026). Looking back, the most neglected stage is often the verification of the solution's reasonableness, checking calculations, reflecting on alternative approaches, and consolidating learning (Schoenfeld, 2016). Verification errors are metacognitive failures in which students accept implausible results, negative volumes, dimensionally incorrect answers, or solutions that contradict problem constraints without critical evaluation (Bray, 2011).

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A study focused on error analysis, including two types of errors: random and systematic (Ryan & Williams, 2007). When we analyse these two types of errors in relation to the system and method of instruction, systematic errors are of more concern than random errors. Not only do systematic errors occur across the board, but they can also be resolved through interventions that directly relate to how students are instructed (Peng & Luo, 2009). Research has demonstrated that students from diverse populations frequently exhibit similar error patterns in their calculus work (Tall, 2013). Common errors when studying calculus include the following: 1) students typically confuse the difference between taking the derivative of a function and taking the integral of a function. 2) Some students treat taking the integral of a function as merely reversing the derivative of a function (Thompson, 2013). This happens when the students do not have enough understanding of the concept. 3) Students tend to apply a pattern of behaviour when calculating integrals. This sometimes happens by using the formula without understanding why it works (Thompson, 2013). Other studies examined errors students make while learning calculus. Rasmussen, Luchies, and Becker (2014) found that students complete the process of using a procedural algorithm. The findings show that most students learn without having any sense of what they have accomplished (Rasmussen, Luchies, & Becker, 2014). When examining the VOSR challenges students encounter, several types of errors are common across most studies. For example, Sealey (2014) found that students frequently misidentify their axis of rotation Sealey (2014). Others confuse their radius and height when using volume calculations. Students experience difficulty translating between verbal descriptions and mathematical representations (Sealey, 2014).

Von Korff and Rebello (2012) studied the strategies students frequently use when selecting an incorrect method for calculating volumes (disk, washer, or shell). Their study provides evidence that the errors associated with VOSR are not merely computational. The researchers illustrated conceptual deficiencies as a major obstacle in their findings. A study by Drijvers et al. (2010) found that technology can be integrated into mathematics instruction in several ways that help reduce errors. One mechanism is offloading. This happens when technology shifts computational demands from working memory to the device. Technology also frees students' cognitive resources for conceptual reasoning and planning (Drijvers et al., 2010). Computer Algebra Systems (CAS) like Maple allow students to see each step of a solution and get immediate feedback. If Maple confirms an answer is correct, the student knows an arithmetic mistake did not occur (Heid & Blume, 2008; Kendal & Stacey, 2001). Research shows CAS use changes the error profile. Computational errors decrease, though errors involving algorithm selection or problem-solving strategy can persist (Ngcobo, 2026).

A study by Hohenwarter and Jones (2007) found that dynamic geometry tools such as GeoGebra add a visual dimension to mathematical representations. The findings show that GeoGebra supports interactive exploration. Hohenwarter and Jones' study also found that GeoGebra helps students develop algebraic thinking, numerical studies and geometric representations of the same concept. This helps students build relational understanding (Hohenwarter & Jones, 2007). Studies report that GeoGebra reduces visualisation and planning errors by letting students see complex relationships concretely (Caligaris et al., 2015; Shadaan & Leong, 2013). In a related study, it was found that verification errors persist even where technology is available. This points to a continuing need for explicit metacognitive instruction regardless of which tool is used (Artigue, 2002). A study found that technology can also introduce new error types, such as syntax errors, misreading of graphical output, or over-reliance on results without conceptual verification (Drijvers et al., 2010). By inference, the use of technology in mathematics does not automatically eliminate every type of error.

The literature on mathematical errors in African education shows both general and context-specific error patterns (Machisi, 2021). Machisi (2021) studied secondary school students in Zimbabwe and identified three causes of systematic error: missing prerequisite knowledge, weak command of mathematical language, and limited exposure to varied problem types. Luneta (2015) found that South African students struggled with algebra in part because they focused narrowly on arriving at an answer rather than understanding the underlying calculations, and that insufficient problem-solving experience compounded the difficulty in calculus. Several studies have examined calculus-specific errors in Ghana. Mereku and Adubofour (2005) identified systematic errors in Ghanaian university students' understanding of limits, derivatives, and integrals. Wilmot et al. (2011) found that Ghanaian students struggled particularly with word problems requiring mathematical modelling. Among pre-service teachers specifically, Anthony and Agyemang (2020) found that high mathematics anxiety increases error rates in abstract content areas like differentiation. Kwarteng et al. (2022) found that while most College of Education graduates report high confidence, few feel equipped to identify and correct errors in their own students' work, a gap that risks perpetuating error patterns across generations of learners.

Research on technology integration in Ghana shows mixed results. Hanaysha et al. (2023) found that technology use improved learning outcomes and classroom participation, but results varied substantially with teacher competence and were less consistent in rural schools with limited resources. Abroampa et al. (2025) found that poor ICT provision often pushes teachers toward personal laptops or low-cost substitutes; while this improvisation supports some creativity, it produces uneven implementation and no consistent reduction in mathematical errors.

The study was guided by the three research questions:

1. What are the differences in error patterns of students using Maple (mathematical software), GeoGebra (Geometric Software) and the traditional approach to the teaching and learning of mathematics?
2. What are the best moments and the worst moments in Pólya's problem-solving framework for the three approaches to the teaching and learning of mathematics?
3. What are the implications of these moment patterns for pre-service teacher education in Colleges of Education in Ghana?

Methodology

This study used a quasi-experimental, pretest/posttest, non-equivalent groups design to examine error patterns across three approaches to teaching the volume of solids of revolution. Three Colleges of Education in Ghana took part: one in the Ashanti Region (urban, well-resourced), one in the Western North Region (semi-urban, moderately resourced), and one in the Bono Region (emerging, resource-constrained). Each college received a single instructional condition: traditional lecture, Maple-enhanced instruction, or GeoGebra-enhanced instruction applied to an intact, pre-existing class rather than through random assignment. This arrangement enables the study to capture instructional effectiveness across a range of resource settings that broadly reflect Ghana's Colleges of Education. But it comes with a real cost: college and instructional conditions are fully confounded. Any difference between groups could come from the teaching method, from something about the specific college, its resources, its student intake, its instructor or from some mix of both, and this design cannot tell those apart. That limitation is taken up further below. Participants were 120 third-year mathematics education students (M age = 21.3 years, SD = 1.8; 62% male, 38% female) drawn from intact classes at three Colleges of Education in Ghana. The three colleges were selected from Ghana's approximately 46 Colleges of Education through purposive sampling to represent a range of resource settings: urban and well-resourced (Ashanti Region), semi-urban with moderate resources (Western North Region), and resource-constrained (Bono Region). Within each participating college, one intact third-year class was assigned to the instructional condition associated with that institution: traditional lecture, Maple-enhanced instruction, or GeoGebra-enhanced instruction (n = 40 per group). Third-year students were selected because they had completed foundational calculus coursework but had not yet studied volumes of solids of revolution, giving the groups comparable baseline exposure to the topic. The three groups showed equivalent prior mathematics achievement based on senior high school final examination scores (Control: M = 65.2, SD = 8.4; Maple: M = 63.8, SD = 9.1; GeoGebra: M = 64.5, SD = 8.7; $F_{2, 117} = 0.42$, $p = .658$). This study received ethical clearance from the Humanities and Social Sciences Research Ethics Committee (HuSSREC) at Kwame Nkrumah University of Science and Technology (Reference: HuSSREC/AP/78/VOL.3). Participation was voluntary: students were informed of the study's purpose, procedures, and their right to withdraw at any point without penalty before providing written informed consent. Given that all participants were enrolled in the participating instructors' classes, students were also informed that non-participation would not affect their course standing to address any perceived pressure to take part. All data were anonymised before analysis, with student identifiers replaced by numeric codes accessible only to the research team; data were stored securely and used solely for this study. All groups received eight weeks of instruction (three 90-minute sessions weekly, 36 total contact hours). Each instructional condition was delivered by a different lecturer at their respective institutions; instructors travelled to their assigned colleges to deliver the sessions. Because the lecturer and instructional conditions were both fixed at the college level, instructor effects cannot be statistically separated from either college effects or treatment effects in this design. Content coverage and instructional pacing were standardised across groups through a shared lesson plan and timetable, with only pedagogical approach and tool use differing by condition. Conceptual understanding, procedural execution, and application were measured using a 15-item researcher-developed test (on a 100-point scale) as a pretest (Week 1) and a posttest (Week 10). The instrument was found to have acceptable reliability (Cronbach's $\alpha = .84$) and content validity was verified by experts.

Results

Pretest Error Patterns: Baseline Challenges

At pretest, all three groups exhibited substantial and remarkably similar error rates across Pólya's problem-solving stages, establishing comparable baseline difficulties before intervention. Verification errors were most prevalent across all groups (77.5% overall), indicating widespread deficiencies in metacognitive monitoring. Execution errors followed closely (70.0%), reflecting computational and procedural challenges. Planning errors (68.3%) suggested difficulties with strategic decision-making, while understanding errors (62.5%) revealed conceptual comprehension gaps.

Table 1: Pretest Error Distribution Across Pólya Problem-Solving Stages by Group

Stages	Control Group	Maple Group	GeoGebra Group	Total (%)
Understanding	27 (67.5)	25 (62.5)	23 (57.5)	75 (62.5)
Planning	26 (65.0)	31 (77.5)	25 (62.5)	82 (68.3)
Execution	28 (70.0)	27 (67.5)	29 (72.5)	84 (70.0)
Verification	33 (82.5)	32 (80.0)	27 (67.5)	93 (77.5)
Total errors	114	115	104	333

Note. Percentages indicate the proportion of students committing errors at each stage.

The similarity in total error counts (Control = 114, Maple = 115, GeoGebra = 104) confirmed that the groups began the study with equivalent problem-solving challenges, validating the effectiveness of the random assignment and establishing a solid baseline for comparing post-intervention changes.

Posttest Error Patterns: Divergent Trajectories

Following the eight-week intervention, dramatic differences emerged in error patterns across instructional groups (Table 2). Technology-enhanced groups demonstrated substantially fewer errors across all stages than the control group, though verification errors remained concerning in all conditions.

Table 2: Posttest Error Distribution Across Pólya Problem-Solving Stages by Group

Stages	Control Group	Maple Group	GeoGebra Group	Total (%)
Understanding	14 (35.0)	3 (7.5)	5 (12.5)	22 (18.3)
Planning	13 (32.5)	6 (15.0)	4 (10.0)	23 (19.2)
Execution	16 (40.0)	4 (10.0)	5 (12.5)	25 (20.8)
Verification	24 (60.0)	12 (30.0)	10 (25.0)	46 (38.3)
Total errors	67	25	24	116

Note. Percentages indicate the proportion of students committing errors at each stage.

The control group demonstrated modest improvement from pretest (114 errors) to posttest (67 errors), representing a 41.2% reduction but leaving substantial error rates across all stages. In stark contrast, technology-enhanced groups achieved dramatic error reductions: Maple decreased from 115 to 25 errors (a 78.3% reduction), while GeoGebra decreased from 104 to 24 errors (a 76.9% reduction). Examining specific stages revealed strengths in the instructional approach. Maple proved particularly effective for understanding errors (from 62.5% to 7.5%) and execution errors (from 67.5% to 10.0%), likely due to its step-by-step computational support and symbolic manipulation capabilities. GeoGebra excelled at reducing planning errors (from 62.5% to 10.0%) and execution errors (from 72.5% to 12.5%), probably reflecting its dynamic visualisation support for method selection. Most concerning, verification errors persisted as the most prevalent problem across all groups despite overall improvements. Even in technology-enhanced conditions, 30% (Maple) and 25% (GeoGebra) of students failed to adequately verify their solutions, compared to 60% in the control group. This pattern suggests that metacognitive monitoring requires explicit instructional emphasis beyond the provision of technological tools.

Error Reduction Analysis

Table 3 presents a detailed error reduction analysis, showing both the number of students who improved at each stage and the percentage reduction in error rates from pretest to posttest.

Table 3: Error Reduction from Pretest to Posttest by Pólya Stage and Instructional Method

Stages	Control Group	Maple Group	GeoGebra Group(%)
Understanding	13 (32.5, 48.1)	22 (55.0, 88.0)	18 (45.0, 78.3)
Planning	13 (32.5, 50.0)	25 (62.5, 80.6)	21 (52.5, 84.0)
Execution	12 (30.0, 42.9)	23 (57.5, 85.2)	24 (60.0, 82.8)
Verification	9 (22.5, 27.3)	20 (50.0, 62.5)	17 (42.5, 63.0)
Overall	47 (29.4)	90 (56.3)	80 (50.0)

Note. Values represent: number of students reducing errors, percentage of group improving, and percentage reduction in error rate (in parentheses).

Several important patterns emerged. First, the technology tools provided comprehensive support across all stages of problem-solving rather than benefiting only specific phases. This contradicts the assumption that CAS primarily aids execution, while dynamic geometry software only enhances visualisation; both tools improved performance across all stages. Second, different tools demonstrated particular strengths. Maple achieved the highest reduction in understanding errors (88.0%) and execution errors (85.2%), suggesting that computational accuracy and step-by-step displays particularly support problem comprehension and procedural execution. GeoGebra demonstrated exceptional effectiveness in addressing planning errors (84.0%) and execution errors (82.8%), indicating that dynamic visualisation facilitates the appropriate selection of methods and accurate implementation. Third, verification remained problematic across all conditions. Even with technology support, error reduction at the verification stage lagged substantially behind other stages (Control: 27.3%; Maple: 62.5%; GeoGebra: 63.0%). This persistent weakness indicates that students fail to develop verification habits spontaneously, even when tools make checking easier. Explicit metacognitive instruction appears necessary regardless of technological affordances.

Specific Error Type Analysis

Beyond stage-based analysis, examination of specific error types revealed distinct patterns across instructional approaches (Table 4).

Table 4: Main Error Types by Instructional Group at Posttest

Conceptual Errors	Control Group	Maple Group	GeoGebra Group (%)
Misidentifying the axis of rotation	18 (45.0)	4 (10.0)	6 (15.0)
Incorrect radius function selection	22 (55.0)	5 (12.5)	7 (17.5)
Formula confusion (disk vs. shell)	16 (40.0)	3 (7.5)	5 (12.5)
Procedural Errors			
Integration-bound errors	14 (35.0)	6 (15.0)	4 (10.0)
Algebraic manipulation mistakes	19 (47.5)	7 (17.5)	8 (20.0)
Integration technique errors	17 (42.5)	4 (10.0)	6 (15.0)
Computational Errors			
Arithmetic calculation mistakes	21 (52.5)	2 (5.0)	9 (22.5)
Unit inconsistencies	13 (32.5)	3 (7.5)	5 (12.5)
Visualization Errors			
Unable to sketch the region	25 (62.5)	8 (20.0)	4 (10.0)
Cannot conceptualise a 3D solid	28 (70.0)	9 (22.5)	5 (12.5)
Misinterpreting cross-sections	20 (50.0)	6 (15.0)	3 (7.5)
Verification Errors			
No answer checking	24 (60.0)	12 (30.0)	10 (25.0)
Ignoring negative volumes	11 (27.5)	2 (5.0)	3 (7.5)

Visualisation errors were most prevalent in the control group, where 70% of students struggled to conceptualise three-dimensional solids, compared with significantly lower rates in the technology groups (Maple: 22.5%; GeoGebra: 12.5%). This finding underscores that static chalkboard diagrams inadequately support spatial reasoning for most students. GeoGebra's interactive rotational displays proved particularly effective, reducing visualisation errors to minimal levels. Conceptual errors, particularly misidentifying rotation axes and selecting incorrect radius functions, affected nearly half of the control students but were dramatically reduced in technology groups. The ability to manipulate visualisations and immediately observe the consequences apparently helped students develop accurate mental models of relationships involving volumes of revolution. Computational errors showed the starkest differences between technology approaches. Maple reduced arithmetic mistakes to nearly zero (5.0%), reflecting its computational automation. GeoGebra users still made occasional calculation errors (22.5%), though substantially fewer than control students (52.5%). This pattern suggests that while visualisation tools aid conceptual understanding, computational support specifically addresses calculation accuracy. Most concerning, verification errors persisted across all groups. Sixty percent of control students failed to check their answers, with technology users performing better but still showing concerning rates (Maple: 30.0%; GeoGebra: 25.0%). Even more troubling, substantial proportions of all groups accepted mathematically impossible results, such as negative volumes, without questioning their validity. This pattern indicates that students lack verification habits and metacognitive monitoring skills that technology alone cannot instil.

Statistical Significance of Error Pattern Differences

Chi-square tests examined whether error distributions across Pólya's stages differed significantly between instructional groups (Table 5).

Table 5: *Chi-Square Tests of Independence for Error Distributions*

Comparison	$\chi^2(\text{df}, N)$	Value	p-value	Effect size
All three groups	18.47	6	.005	.45
Control vs. Technology combined	15.23	3	.002	.58
Maple vs. GeoGebra	1.83	3	.607	.35

Note. The combined group includes both the Maple and GeoGebra groups ($n = 80$).

The chi-square analysis indicated a significant difference in error distribution across instructional groups, $\chi^2(3, N = 116) = 15.23$, $p = .005$, Cramer's $V = .45$, a large effect. This is consistent with the instructional approach being associated with the problem-solving stages producing the most errors. The most striking finding emerged when comparing traditional instruction to technology-enhanced approaches combined, $\chi^2(3, N = 116) = 15.23$, $p = .002$, Cramer's $V = .58$. This large effect size demonstrates that technology integration fundamentally altered error patterns rather than merely reducing overall error frequencies. Control group students exhibited relatively evenly distributed errors across all four stages, suggesting persistent difficulties throughout the problem-solving process. Technology users showed concentrated improvements in understanding and execution stages but continued to struggle with verification, revealing a distinct problem-solving profile. Interestingly, no significant difference emerged between Maple and GeoGebra, $\chi^2(3, N = 80) = 1.83$, $p = .607$, Cramer's $V = .35$. Despite their different technological affordances, computational support versus dynamic visualisation, both tools produced remarkably similar error distributions across problem-solving stages. This finding suggests that the presence of technology, rather than specific tool characteristics, is the critical factor influencing error patterns.

Discussion

Technology's Differential Effects on Error Types

This study's systematic error analysis reveals nuanced insights into how technology-enhanced instruction influences mathematical problem-solving that aggregate achievement scores alone cannot provide. While both Maple and GeoGebra significantly improved overall performance, examining specific error patterns illuminates the mechanisms underlying these improvements and identifies persistent challenges requiring additional pedagogical attention.

The reduction in understanding errors was larger for the technology groups (88.0% for Maple, 78.3% for GeoGebra) than for traditional instruction (48.1%). This pattern is consistent with technology supporting problem comprehension, though the non-randomised, college-level assignment and unmodeled clustering mean the data cannot establish that technology

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caused this difference on its own. Interactive visualisations may help students build more accurate mental models of volume-of-revolution concepts, turning abstract symbolic representations into objects students can manipulate directly. This is consistent with cognitive load theory (Sweller, Ayres, & Kalyuga, 2011): external representations that students can explore may reduce the burden of holding a visualisation purely in working memory, leaving more capacity for conceptual reasoning.

Similarly, substantial reductions in execution errors reflect the computational support of technology. Maple's near-elimination of arithmetic mistakes (85.2% reduction) confirms that offloading calculations to technology allows students to focus on strategic thinking rather than procedural mechanics. Even GeoGebra, which offers less computational automation, achieved an 82.8% reduction in execution errors by enabling students to verify calculations visually and catch mistakes before finalising solutions. However, the persistence of verification errors (60% control, 30% Maple, 25% GeoGebra) reveals the technology's limitations. Despite tools that make checking easier, Maple's symbolic verification capabilities and GeoGebra's visual reasonableness checks, substantial proportions of students simply failed to verify their work. This pattern suggests that verification represents a metacognitive habit and dispositional orientation rather than merely a skill that technology can scaffold. Students must develop the inclination to question results, the habit of checking reasonableness, and the persistence to investigate discrepancies, qualities that require explicit cultivation regardless of available tools.

Implications for Pólya's Framework in Technology-Enhanced Contexts

This study extends Pólya's problem-solving framework by demonstrating how technology differentially supports its four stages. Traditional discussions of Pólya's model assume human-only problem-solving; this research reveals how technological tools restructure problem-solving processes. Understanding and planning stages, which Pólya characterised as requiring insight, creativity, and strategic thinking, benefit substantially from technology's visualisation and exploration capabilities. Dynamic representations transform the "understanding the problem" stage from passive interpretation of static text and diagrams to active exploration, in which students manipulate problem elements to construct meaning. Similarly, "devising a plan" becomes experimentally grounded: students can test different methods, observe consequences, and develop informed strategies rather than relying purely on abstract reasoning. The "carrying out the plan" stage fundamentally changes with technology. Pólya emphasised careful execution and computational accuracy; technology eliminates this concern, enabling focus on conceptual coherence rather than algebraic mechanics. This shift potentially represents profound pedagogical improvement: execution errors often obscure whether students understand concepts or merely make calculation mistakes. Technology clarifies this ambiguity. However, "looking back," Pólya's fourth stage remains underdeveloped even with technological support. This finding highlights that verification requires explicit instructional emphasis. Technology provides tools for checking, but cannot instil the disposition to use them. Metacognitive instruction teaching students when and how to verify, why verification matters, and what constitutes reasonable results appears necessary regardless of technological affordances.

Error Patterns and Teacher Education

For Ghana's Colleges of Education specifically, these findings are particularly significant. Pre-service teachers' own error patterns are likely to shape their future teaching effectiveness: teachers who do not recognise an error in their own work are poorly positioned to diagnose it in a student's work (Ball et al., 2008). The persistent verification errors observed across all instructional groups suggest that explicit metacognitive instruction may be worth integrating into teacher education programs, though this study did not directly test such an intervention. Understanding which errors technology appears to address, and which persist regardless of method, may help pre-service teachers form more realistic expectations of technology's pedagogical role. Technology functioned here as a support tool rather than a complete solution. Effective integration likely requires teachers who understand both what these tools can do well (visualisation, computational accuracy, immediate feedback) and what they did not resolve in this study (verification behaviour and, to a lesser extent, conceptual reasoning at the planning stage).

Maple and GeoGebra produced broadly similar error-reduction profiles despite differing in what they make visible to students, which is consistent with some flexibility in how technology-based instruction can be implemented, though this study cannot say whether that similarity would hold with different instructors, longer exposure, or other content areas. Several features of the design limit the extent to which these results can be generalised. First, each instructional condition was delivered at a separate intact classroom, meaning college/site is confounded with treatment; differences attributed to

method cannot be fully separated from differences between the three institutions or instructors. Second, where a single instructor delivered each condition, instructor effects cannot be distinguished from treatment effects. Third, the eight-week intervention window establishes short-term gains but says nothing about whether these differences persist; no follow-up assessment was conducted to test durability. Fourth, error coding of student work was conducted by researchers without a reported inter-rater reliability statistic (e.g., Cohen's κ), so the reliability of the error classification itself is not established. Fifth, the sample was drawn from three of roughly 46 Colleges of Education in Ghana, all within a limited geographic area; results may not generalise to private universities, other regions, or institutions with different resource levels. Finally, students in the technology conditions were using Maple and GeoGebra for instructional purposes for the first time in this context, and the study did not control for novelty effects. Some portion of the technology groups' gains may reflect heightened engagement with an unfamiliar tool rather than a durable instructional advantage of the software itself. These limitations do not undermine the pattern of results, but they do mean the findings should be read as evidence from a specific, constrained context rather than as a general claim about technology's effect on calculus instruction.

Conclusion

The following three research questions were raised in this study: (RQ1) What are the differences in error patterns of students using Maple (mathematical software), GeoGebra (Geometric Software) and the traditional approach to the teaching and learning of mathematics? (RQ2) What are the best moments and the worst moments in Pólya's problem-solving framework for the three approaches to the teaching and learning of mathematics? (RQ3) What are the implications of these moment patterns for pre-service teacher education in Colleges of Education in Ghana?

Error patterns of different teaching methods. The technology groups experienced far fewer understanding and execution errors, and the groups that used GeoGebra and Maple had comparable execution error rates, both at 25%. The number of errors in understanding and execution was quite low in the technology groups. However, it was high across all three conditions: control (60%), Maple (30%), and GeoGebra (25%). In the context of adopting technology, the advantages of a given technology over another should not be assumed. The profiles of error reduction for the two technologies were not dissimilar, and the two technologies will afford different possibilities for computation and visualisation, so that institutions with budget or infrastructure constraints might use one or the other (if implemented coherently and purposefully) and might not expect much difference.

How are differences in the effects at Pólya's stages recorded? The use of technology proved very helpful in visualising, planning, and implementing the stages, but was not very helpful for the "looking back" (verification) stage, with or without the instructional method. This finding reveals that verification is not limited to the scaffolding of the technology but also to a metacognitive disposition. Technology, therefore, has to be used in instruction as a facilitating tool for the first three steps of problem-solving, not for the fourth step. The verification gap must be closed through deliberate instructional planning, not by adding features to the software.

What implications does this have for teacher education? The high percentage of verification errors across all groups may indicate that pre-service teachers have unexamined error patterns that interfere with their ability to later identify similar errors made by their students (Ball et al., 2008), thus representing a gap in pre-service teacher training that can be addressed. Based on the findings, it is recommended that the Colleges of Education provide explicit metacognitive instruction in combination with technology use in teaching calculus and continue to provide such instruction without technology and (b) reflective tasks in college courses which require the pre-service teacher to directly analyse his/her own errors in order to better understand and identify similar error patterns in his/her students.

The results of the combined studies indicated that at certain stages of problem solving, technology could play a significant role in pre-service teachers' calculus problem-solving performance, whereas at other stages. It should not be used as a substitute for explicit metacognitive teaching. Future research is needed that involves random allocation, interrater reliability checks for error coding, and post-testing to determine whether the patterns observed during these eight weeks at these particular institutions can be applied to other institutions and longer time periods.

Recommendations

Based on the results of the study, the recommendations are as follows:

1. Metacognitive Instruction: Colleges of Education should purposefully include overt metacognitive instruction when teaching mathematics with and without technology to fill the verification gap.
2. Reflective Practice: Ask Pre-service teachers to reflect on and critique their own errors so they can more easily spot and fix similar patterns of errors with students.
3. Purposeful Technology Integration: Adopt Maple or GeoGebra with purpose and consistency. Both technologies offer equal opportunities for computation and visualisation.
4. Instructional Design: Allow technology to support the first three stages of problem solving and strengthen verification through careful instructional design vs software capabilities.
5. Future Research: Randomly assign participants, include interrater reliability, and provide a longer-term post-test to see if results are generalizable.

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